SPECIES SEPARATION IN NARROW GAPS–A SIMPLE MODEL

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Abstract. The problem of the separation of Uranium isotopes from the fluid-dynamic point of view is considered. The technique based on the separator nozzle effects is discussed. As this system works at low pressure, the analysis is performed in the slip flow regime. The boundary layer, transition and asymptotic regions for the velocity and concentration profiles in the nozzle are studied.

Keywords. Entrance flow in ducts; slip flow; separation nozzles.

INTRODUCTION

Isotope ^{235}U, fissionable by thermal neutron, is present in natural uranium as 0.7 mole %, while 2% is required for fusion. Static diffusion and the gas centrifuge process are the most important ^{235}U enrichment systems.

An alternative method is based on separation nozzle which rely on pressure gradients and inertial forces in a curved flow configuration for separating uranium isotopes from a gas containing UF_6.

In this respect, the first tentative experiments, suggested by Dirac 50 years ago, were not promising. Becker, 20 years ago, used successfully, instead of pure UF_6, a mixture of UF_6 and light auxiliary gas in high molar excess (Fig.1).

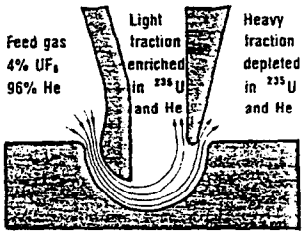

Fig. 1

The complicated flow in a separation nozzle has only partly been assessed theoretically. In Ref. |1| some preliminary results are discussed. Purpose of the present analysis is to present some fluid-dynamic models in the different regions of the nozzle for some parameters of the problem.

The mixture considered here UF_6 + He. Uranium hexafluoride under ordinary conditions is a dense white solid with a vapour pressure of about 120 mm Hg at room temperature. Despite its high molecular weight gaseous UF_6 is almost a perfect gas and many of the properties of the vapour can be predicted by the kinetic theory of the gases.

A separation nozzle works at low pressure (typical value: 30 mb). The Knudsen number

$$k = \frac{\text{mean free path of the molecules}}{\text{height of the nozzle}}$$

is in the range 0.01–0.1 and, therefore, the flow occurs in the slip regime. The velocity at the wall is then related to the slope of the velocity profile by the condition $u_w = k\, u_{n,w}$.

In the nozzle three kinds of behaviour of the velocity and concentration profiles are possible: boundary layer, transition and fully developed flows according to a scheme usually adopted in the entrance problem for a channel. (See upper corner of Fig. 2).

In this analysis to calculate the two abscissas x_{BL} (BOUNDARY LENGTH) and x_{TR} (TRANSITION LENGTH) characterizing the three regions for the present problem, approximate solutions for the velocity and the concentration fields are carried out. One considers here the influence of the Knudsen number k, the velocity U and the curvature 1/r of the inner surface, the Reynolds and Schmidt numbers, the molecular weight of the mixture and the initial conditions in simple models.

To solve the two diffusion equations it is assumed that the diffusion of the light isotope in the mixture can be treated as the diffusion in a uniform gas; therefore the diffusion coefficient is approximately given by

$$1/D = N_h/D_{l,h} + (1-N_h)/D_{a,u}$$

l = light, h = heavy, a = auxiliary. Moreover one assumes that the diffusion of the heavy isotope in the mixture is not influenced by the light isotope.

VELOCITY PROFILES

A) Boundary Layer Region

The basic equations are

$$u_x + v_y = 0 \; ; \qquad uu_x + vu_y = u_{yy}/R$$

$$y = 0 : u = ku_y + U \; ; \; y = 1 : u = -ku_y$$

This problem has been solved by an extension of the method of Ref. |2|. It results

$$u = 1 + \frac{U-1}{k/h+1} \exp(-y/hR^{1/2})$$

where

$$4x = (1+U)(2kh+h^2) + 2k^2(1-3U)\log(1+h/k) + -(1-U)k^2h/(h+k)$$

If one supposes that the boundary layer region ends when the argument of the exponential is equal to -4 at y = 1/2, then the abscissa x_{BL}, given by the condition $h = 1/8R^{1/2}$, is

$$4x_{BL} = (1+U)(2k/8R^{1/2}+1/64R)+2k^2(1-3U) \log(1+1/8hR^{1/2})-4k^2(1-U)/(8kR^{1/2}+1)$$

and varies linearly with U. In particular, for U = 0 and k = 0 one has $x_{BL} = 1/256R$.

B) Fully developed profiles

When u does not depend on x it is

$$u = 6[k+y(1-y)]/(1+6k) + U(1+k)/(1+2k)$$

The reference velocity is the initial mean one.

C) Transition region

It is now

$$uu_z + vu_y + p_x = u_{yy} \; ; \; u_z + v_y = 0 \qquad z = sx$$

with the b.c.: y=0 $u=ku_y+U$; y=1 $u=-ku_y$

Let: $u = u_o + u_1$, where u_o is the asymptotic velocity.

We assume $u_1 = a(z)(a_o + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4)$ where the coefficients $a_i(k,U)$ are determined through the boundary conditions. The function a(z) and p_x are determined through the momentum and continuity equations integrated between 0 and 1, and evaluated at y=0.5.

It results a = Aexp(-bz), and if one assumes that the transition region ends at bz = 4, i.e. for z = 4/b, then

$$z_{TR} = 4/b \; ; \; x_{TR} = 4/sb$$

Figures 2, 3, 4 show the main results of the calculations. There L_{BL} and L_{TR} represent the lengths of the boundary layer region and of the transition region, respectively.

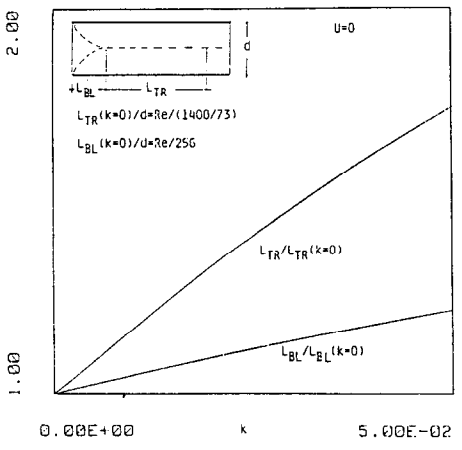

Fig. 2

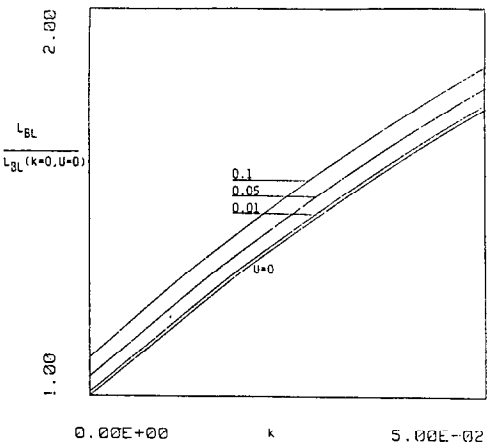

Fig. 3

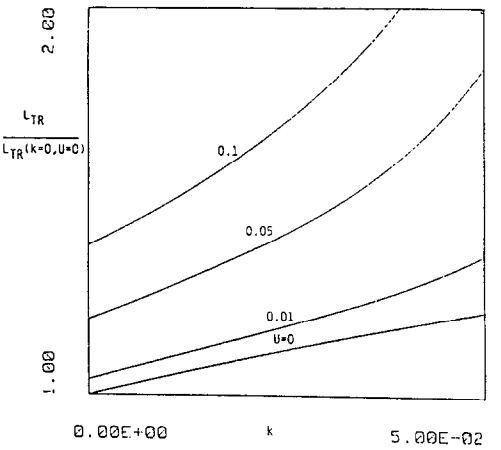

Fig. 4

CONCENTRATION PROFILES

A) Boundary layer region

The governing equation for the concentration is

$$c_z = (c_y + 2Ac)_y$$

where

$$2A = (p_{mi} - p_{mj})p_y / pp_m \; ; \; z = sx \; ; \; s = D/Ud$$

with D = diffusion coefficient; p_m = molecular weight, p = pressure.

The boundary conditions are: $c(0,y) = c(z,\infty) = 1$; $c_y(z,0) = -2Ac(z,0)$.

This problem has been solved by means of the Laplace Transform technique and one has

$$c = 1 - A(y/4 - 7/8A - Az/2)\exp(-2Ay)\operatorname{erfc}(y/2z^{1/2} + -Az^{1/2}) + (7/8)\operatorname{erfc}(y/2z^{1/2} + Az^{1/2}) + (Az^{1/2}/2^{1/2}) \cdot \exp-(y/2z^{1/2} + Az^{1/2})^2.$$

Again if one assumes that the boundary layer region ends when the smallest argument of the error function is equal to 2 at y = 1/2, then the abscissa x_{BL} is given by

$z^{1/2} = [(1+A/4)^{1/2}-1]/A$

which, for $A < 1$, gives the simpler approximate expression $x_{BL} = 1/64\varepsilon$.

For the mixture $^{238}UF_6$ + He (5 mole + 95 mole) it is $(p_{mi}-p_{mj})/p_m = 16.26$. At p = 30 mb, say d=1 mm, u = 250 m/sec it is D = 26 cm^2/sec, s = 0.01, A = 4.35 d/r.

Figure 5 shows the behaviour of the boundary layer thickness δ_c as a function of x, δ_c being defined according to the 99% rule.

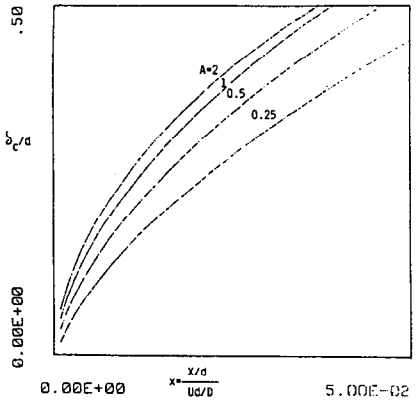

Fig. 5

B) Asymptotic profile

An asymptotic solution, independent of x, can be found from the diffusion equations, by imposing that the diffusion flux vanishes. This simple solution can be obtained directly from the equilibrium condition:

$p_{ir} = \rho_i u^2/r$

where p_i and ρ_i are the partial pressure and the partial density respectively, taking into account that $p_i = \rho_i R_o T/p_{mi}$. As a result

$$\rho_i/\rho_{i1} = \exp\left(\frac{p_{mi}}{R_o T}\int_{r_1}^{r} u^2 dr/r\right)$$

For $u = \omega r$, with ω constant angular velocity, it is

$$\rho_{i1} = \langle\rho_i\rangle(1-p_{mi}\omega^2 r_2^2 a_1/R_o T)\ ,$$

$$\rho_{i2} = \langle\rho_i\rangle(1+p_{mi}\omega^2 r_2^2 a_2/R_o T)$$

where a_1 and a_2 are numerical coefficients of the order of magnitude 1 and $\langle\rho_i\rangle$ is the mean value of ρ_i.

To obtain an appreciable difference between the values of ρ_i at $r = r_1$ and at $r = r_2$, high values of ω are required, or more in general high values of

$$\int_{r_1}^{r_2} u^2 dr/r$$

The linearized solution is $c = [2Aexp(-2Ay)]/(1-exp-2A)$.

C) Transition region

In this region the diffusion equation is

$c_z = (c_y+2Ac)_y$

with $z = sx$; $s = D/ud$ and the b.c. are

$c_y = -2Ac$ at y=0 and y=1 at x=0 c=1.

The problem can be analysed by means of the Laplace Tranform $C = L[c]$ and one has

$C = k_1 expa_1 y + k_2 expa_2 y + 1/t$

where

$a_1 = -A-(A^2+t)^{1/2}$; $a_2 = -A+(A^2+t)^{1/2}$

$k_1 = (expa_2-1)/(a_1+2A)(expa_2-expa_1)$

$k_2 = (1-expa_1)/(a_2+2A)(expa_2-expa_1)$.

The asymptotic solution c can be recovered by passing to $\lim_{t\to 0} tC$. One has again

$c = 2Aexp(-2Ay)/\ 1-exp(-2A)$

The solution c can be written as $c=c+c_1$; thus it is $C_1 = C-c/t$. The properties of the Laplace Transform lead to

$$c_1 = \int_0^z f(z,y)\exp(-A^2 z)dz$$

where F=L[f] is given by

$$F = \frac{AexpA(1-y)}{Sen\ t^{1/2}}\left[\frac{1-exp(t^{1/2}-A)}{A-t^{1/2}}\exp(-yt^{1/2}) + \frac{exp(-A-t^{1/2})-1}{A+t^{1/2}}\exp(yt^{1/2})\right]-c$$

Then: $z_{TR} \leq 4/A^2$; $x_{TR} \leq 4/A^2 s$ and the transition abscissa can be evaluated by writing $c=f(y)\exp(-a^2 z)$.

The diffusion equation and related boundary conditions give

$-a^2 f = f'' + 2Af'$

$f_y = -2Af \qquad y = 0\ ,\ y = 1$

$f = B_1 exp(a_1 y) + B_2 exp(a_2 y)$

$a_{1,2} = -A \pm (A^2-a^2)^{1/2}$

and the boundary conditions lead to

$B_1(a_1+2A) + B_2(a_2+2A) = 0$

$B_1(a_1+2A)expa_1 + B_2(a_2+2A)expa_2 = 0$

The eigenvalues of this system are $a^2 = A^2+n^2\pi^2$ and finally one has

$z_{TR} = 4/(A^2+\pi^2)^{1/2}$; $x = 4/s(A^2+\pi^2)^{1/2}$

Figure 6 shows the concentration profiles at various stations in the transition region.

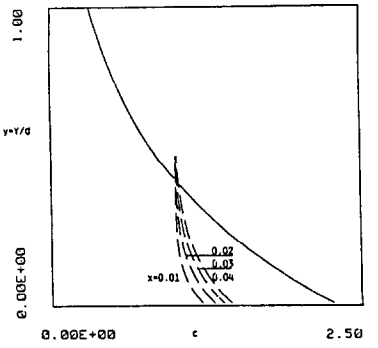

Fig. 6

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